

MATHEMATICS IN THE ISLAMIC SCIENTIFIC TRADITION:

a discourse on the philosophy of mathematics¹

Prof. Dr. Mulyadhi Kartenegro

I. PROLOGUE

Like other disciplines, mathematics, in the modern times, has long been divorced from its mother: philosophy or metaphysics. Consequently, mathematics that used to have a clear intermediary position between physics and metaphysics, now it has lost its secure place. Scholars have quarreled as to whether it is science or simply instrument of science.

It is for this reason, among the others, that the majority of contemporary scholars in mathematics do not pay sufficient attention to the philosophical issues closely related, especially in the classical periods, to mathematics. Consequently, although these issues are very important for our understanding more deeply of mathematics, most of us now have no idea about them.

It is for this reason I would like to discuss these (philosophical) issues in mathematics, as practiced in the Islamic scientific tradition, as follow:

1. the nature of mathematics,
2. the instrumental role of mathematics,
3. the position of mathematics in the Islamic classification of science,
4. the ontological status of mathematical objects
5. the correspondence between mathematical objects and the external realities.
6. symbolic meanings of mathematics
7. the relation between mathematics and moral Refinement.

I. THE NATURE OF MATHEMATICS

Mathematics has been considered by Muslim scholars as a science that lies between metaphysical and physical sciences. It is viewed as “immaterial in itself but still has some relation to the physical worlds.” This is of course different from physics which always relates with material substance and movement, on one hand, and metaphysics which does not have any direct relation with physical entities.

II. SCIENTIFIC STATUS OF MATHEMATICS

One of the most fundamental questions to mathematics is “Is mathematics a science or not? Any Science that wants to be considered “scientific” in the modern scientific tradition should be “empirical” in nature. Any non-empirical body of knowledge

¹ This paper is presented in the International seminar on Mathematics in UIN Jakarta, October 28, 2008, organized by the faculty of Tarbiah and Education UIN Jakarta.

cannot be considered as science. Our question, now, is “is mathematics a science?” Many scholars consider mathematics as science, hence the term “mathematical sciences.” If the main condition for being accepted as science is “empirical” my question is “what is so empirical about mathematics?” Of course we know that mathematics in itself is not empirical, but why there are some scholars who consider it as science. Mathematics will never meet this fundamental condition of modern scientific criteria: being empirical. Therefore, the scientific nature of mathematics is still a problem.

One day I asked Prof. Dr. Muslim, professor on physics at UGM, if mathematics is considered a science by modern scientists? The answer that I got after several days was that mathematics is not science, but an instrument of science. The reason why mathematics cannot be considered by modern scientists as science is because mathematics is not empirical. Only empirical knowledge can become science. Mathematics, in itself immaterial, cannot be vied as science. Now, the question is why a non-empirical knowledge cannot be considered as science? The answer is because modern scientist views any non-empirical object as not real. Anything that is not empirical is considered non-sense, and hence has no reality.

In the Islamic scientific tradition, however, mathematics has been considered as science, that has a legitimate scientific status like that of physics or astronomy. This is so, because Muslim scholars believe in the ontological reality of not only physical objects (*al-mahsusat*) but also non-physical objects (*al-ma'qulat*), such as mathematical objects. Therefore, Muslim scholars do not have reasons to deny the legitimate scientific status of the mathematics.

III. THE INSTRUMENTAL ROLE OF MATHEMATICS

Now, if mathematics cannot be considered as science, what is, then, its scientific status? Prof. Muslim of Yogyakarta said that mathematics is just a tool or instrument (*alat*) for science. For example, it is still used for studying and understanding of physical entities. In the Islamic scientific tradition, as mentioned above, mathematics was considered as a science, but at the same time, like in the West, as a scientific tool. In astronomy, mathematics has been used by Muslim scientists as scientific method together with demonstrative methods. Mathematics was also used very often to help scientific research in geology and geography. Al-Biruni (d. 1039), for example, used trigonometry when he conducted a geological research for measuring the circumference of the earth. According to Prof. Osman Bakar, mathematics was utilized by Muslim scientists as indispensable instrument for understanding physical matters, such as done by modern physicists.

In addition, mathematics also functioned as a methodological instrument (*alat*) for understanding philosophy, without which philosophy, according to al-Kindi (d. 866), will never be understood properly. Even he wrote a book entitled *Philosophy Cannot be Understood Without Mathematics* to show us how important mathematics is for understanding philosophy. He also says that scientific method can be demonstrated clearly only in mathematics. So according to Muslim scholars, mathematics is a legitimate theoretical science, with its clear theoretical status. Mathematics has also functioned as scientific instrument for other sciences such as physics, geology,

astronomy and even philosophy. Its instrumental role in philosophy can be understood by the fact that mathematics is situated between physics and metaphysics. Although closer to physics, mathematics is not physical science. It is abstracted from material entities, but not entirely separated from matter like metaphysics.

IV. POSITION OF MATHEMATICS IN THE ISLAMIC CLASSIFICATION OF KNOWLEDGE

Unlike in the modern sciences, science in the Islamic scientific tradition has its own hierarchy or gradation. In the West sciences are put randomly at one and the same par. A particular science will find its own proper place in that classification of science. This classification has been deemed very important by Muslim scientists so that almost all the great philosophers and scientists wrote something about it. Mathematics has traditionally been included among the rational sciences by Muslim scientists. And when they classified the rational sciences into theoretical and practical, mathematics belongs to theoretical sciences. These theoretical sciences, in turn, were divided into farther sub-divisions: physics, mathematics and metaphysics, while practical sciences into ethics, economics and politics. It is clear from here that mathematics occupies a middle position between physics and metaphysics. It is in itself immaterial, but it still has connection to the physical world. Metaphysics on the other hand, has no direct relation to the physical world.

Mathematics has been classified into four main divisions, well-known as *quadrivium*: arithmetic, geometry, astronomy and music. Arithmetic deals with number, geometry with spatial dimension and geometric forms, astronomy with the observation, measuring and mapping of the stars, planets and spheres, and music with melody, tones, and harmony. Of course, there are some sub-divisions of these divisions for each subject, such as algebra, calculus, for arithmetic, optics and astronomy for geometry, judicial astrology and mathematical astronomy, for astronomy.

V. THE ONTOLOGICAL STATUS OF MATHEMATICAL OBJECTS

In the modern times, people barely enquire into the ontological status of mathematical objects, whether they are real or just imagination. Most of us think that mathematical objects exist only in our mind, while in the external world there is nothing that we can call mathematical objects or realities. They are purely mental phenomena.

As mentioned above, Muslim scientists and philosophers firmly believed that mathematical objects have their own realities outside of mind. It is true that they are non-physical, so we cannot see them. But, according to them, physical or sensible objects are not the only reality in this world. Some objects are non-physical or intelligible. It is for this reason that they divided the objects of their enquiries into the sensible objects (*al-mahsusat*) and the intelligible objects (*al-ma'qulat*). Mathematical objects, of course, do not belong to the former, but to the later. The proof that mathematics is not physical can be demonstrated by the fact that in mathematics we can state that something can be multiplied or divided infinitely, while in the physical world there nothing that we can multiply or divide infinitely.

In the Islamic scientific tradition, the mathematical objects, like the physical and metaphysical objects, are viewed as real, in the sense having firm ontological status or reality. Even for some philosophers, they are more real than the physical ones. According to Suhrawardi al-Maqtul (d. 1191), the founder of Ishraqi school, mathematical objects as “Platonic ideas,” belong to the metaphysical world. Not only that, he even calls them lights or angels in the horizontal order of light, as the lords of idols (*arbab al-asnam*). By calling them angels, he really wants to demonstrate that mathematical objects are ontologically as real as angels.

But as mathematics, like other non-physical sciences, was secularized in the modern era, the reality of mathematical objects was denied. The modern scientists believe only in the sensible objects, therefore, they reject the ontological status of mathematical objects. Nothing really exists, according to them, save the physical-empirical objects. Because of that they also, as mentioned above, deny the scientific status of mathematics. Therefore, mathematics is no science. It is the tool of science. While in the Islamic scientific tradition, mathematics is not just instrument of science, but also science itself.

VI. THE CORRESPONDENCE BETWEEN MATHEMATICAL OBJECTS AND THE EXTERNAL REALITIES

If mathematics has no ontological contents, it necessarily means that it cannot in any way correspond to the external realities. But if it were so, why is it that mathematics has been utilized up to now in physical, biological and astronomical investigations? This, to me, clearly shows that, whether you like it or not, mathematical concepts do correspond to the external realities. This, in turn, demonstrates its ontological realities. To take as example, we are now definitely sure that we can predict a total solar eclipse several years in advance. Of course, we do it by a mathematical calculation, measurement, or even model. My question is can the prediction come true, if the external world itself does not operate mathematically. Of course not! Thus, it is clear to me that mathematical concepts that exist in our mind do correspond to the mathematical-like operation of the external world.

Moreover, the geometrical forms or shapes do exist in the external world. Spherical forms, for example, can be seen so easily in the natural shapes, such as that of the moon or the Sun. Hexagonal form can be so perfectly found in the bee-hives. As for circular form, we can create it so easily by throwing away a small stone to the lake surface. It will immediately create circular movement of the wave. All these are presented so abundantly before our eyes. If we could look more deeply into sub-atomic particles, the geometrical forms and patterns will be more clearly demonstrated. Therefore, the geometrical forms in our mind can find their counterparts in the physical world. It is why, many Muslim scholars said that we have learned mathematics from the mathematically operated external world. Since mathematics do correspond to the external world, it helps us greatly understand the nature of physical objects under investigation. It is, therefore, understood why most of the scientists from the classical to the modern eras have used mathematics in their scientific research or investigation.

VII. SYMBOLIC MEANING OF MATHEMATICS

In the modern era, mathematics has lost its non-physical aspects. Therefore, it is like an empty frame, without contents. In the Islamic tradition, however, as also in other Oriental tradition, every letter or character has its own numeric values. The numeric value of “alif”, for example, is 1, “ba’,” is 2, “ta’” is 400, and “tha’” is 500. So also every number has its own symbolic meaning. For instance, 1, according to Ikhwan al-Shafa’, symbolizes God, the One and origin of everything and its returning place. They say, “Number is divided into two, solid and fraction. One that exists before two, is the root or basis of number, from which all numbers, solid or fraction, come, and to which every number returns.” Meanwhile, 4 refers to 4 elements, 4 seasons, 4 humors, and 4 directions.”

Moreover, Muslim mathematicians believed that the numeric order is not arbitrary, but corresponds to the natural order. This is because, according to them, most of the natural order of number has been determined by God himself, the Creator. In Ikhwan al-Shafa’ view, “the relation of God to other existences is like that of 1 to other numbers, and the relation of the universal or active intellect to other existences is like that of 2 to other numbers. The relation of the universal soul to other existences is like that of 3 to other numbers, and relation of the prime matter to other existences is like that of number 4 to other numbers.”

Therefore, it is clear to us, that in view of Muslim scholars number has meanings more transcendent than simply an empty frame. And this is one of other things that makes mathematics in the Islamic scientific traditions differs significantly with the modern one.

VIII. THE RELATION BETWEEN MATHEMATICS AND MORAL REFINEMENT

The last feature that I would like to mention here is the fact that mathematics, in Islamic scientific tradition, is quite often connected to the refinement of characters (*tahdhib al-akhlaq*). Of course, this phenomenon is very strange to our contemporary scholars. Miskawayh (d. 1010), in his *Tahdhib al-Akhlaq*, advises us to teach and educate our children with mathematics. It is because the teaching of mathematics will implant into our children hearts love for the truth, and hatred for falsehood. Children will know with certainty that $2 + 2 = 4$. This is the truth. The otherwise will be false. If we train them with this mathematical truth, it will imbed so strongly in their mind and heart, so that they will surely reject the otherwise. They will not tolerate or compromise that, for example, $2+2 = 5$ Or 6, as might happen when we make “the project budget.”

It is also quite significant, to me, that Ikhwan al-Shafa’ in their work, *Rasa’il*, try to connect arithmetic and geometry with the refinement of characters (*tahdhib al-akhlaq*) as obvious from one of sub-titles called “the relation between arithmetic and geometry and moral refinement.” In their discussion on arithmetical and geometrical proportion, they state that “if you put a part of our body on the other parts proportionally, their figure will be good and acceptable, and their characters will be praiseworthy. But if the case is the opposite, their posture will be awkward and frightening and their characters are blame worthy. In other world, arithmetical and

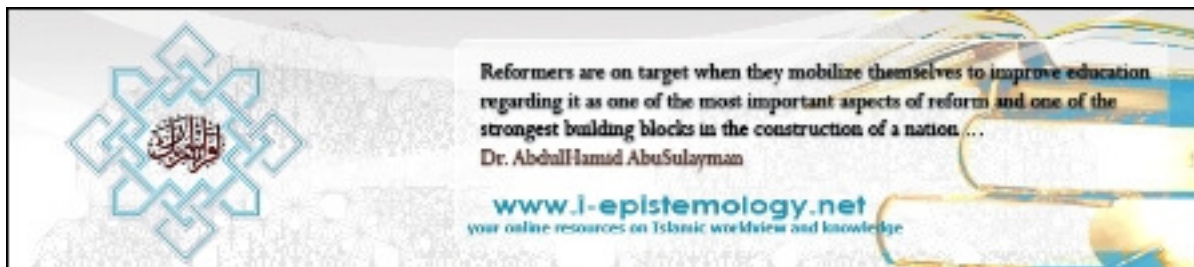
geometrical proportion (*nisbat*) can influence not only physical structure but also human spirituality and morality.

IX EPILOGUE

To conclude, we can learn so much from Muslim philosophers and scientists concerning the philosophical dimensions of mathematics. Their beliefs in the ontological status of mathematical objects help us understand why and how the philosophers believed in the unseen world. Their discussion on the symbolic meanings of numbers, letters and geometrical forms can give us more insights on how nature operates not only in physical, but also in mathematical and metaphysical planes. Their discussion on the correspondence between mathematical concepts and the external world helps us understand why, in spite of its immateriality, mathematics is still used by scientist, both classical and modern ones, in their investigation on natural phenomena.

In short, there are so many crucial insights that the Islamic scientific tradition has contributed to our better and more profound understanding on mathematics. And this I believe can, at least partially, answer to our fundamental question “what is the difference in studying science in Islamic universities and in the public/secular universities?”

Thank you. God bless you!



<http://www.i-epistemology.net/mulyadhi-kartanegara.html>